**Bachelor of Science (B.Sc.) Semester—IV (C.B.S.) Examination**

**MATHEMATICS ($M_8$–Mechanics)**

**Paper—II**



Time : Three Hours] [Maximum Marks : 60

**N.B. :—** (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.



**UNIT—I**

1. (A) If six forces of relative magnitudes 1, 2, 3, 4, 5 and 6 act along the sides of a regular hexagon taken in order, show that the single equivalent force is of relative magnitude 6 and that it acts along a line parallel to the force 5 at a distance from the centre of the hexagon 3½ times the distance of a side from the centre. 6

(B) Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. If a weight W be attached to C and the system be suspended from A, show that there is a thrust BD equal to $W/\sqrt{3}$. 6



**OR**

(C) Derive the Cartesian equation of a common catenary in the form

$$y = c \cosh (x/c).$$ 6

(D) A uniform chain of length $\ell$ is to be suspended from two points A and B in the same horizontal line so that the tension at the high point is twice that at the lowest point. Show that the span is :

$$\frac{\ell}{\sqrt{3}} \log (2 + \sqrt{3}).$$ 6

**UNIT—II**

2. (A) The velocity of a particle along and perpendicular to a radius vector from a fixed origin are $\lambda r^2$ and $\mu\theta^2$. Show that the equation to the path is $\frac{\lambda}{\theta} = \frac{\mu}{2r} + c$ and the components of acceleration are $2\lambda^2 r^3 - \mu^2 \frac{\theta^4}{r}$ and $\lambda\mu r\theta^2 + 2\mu^2 \frac{\theta^3}{r}$. 6

(B) An insect crawls at a constant rate u along the spoke of a cart wheel of radius a, the cart is moving with velocity $v$. Find the acceleration along and perpendicular to the spoke. 6

**OR**

(C) At the ends of three successive seconds the distances of a point moving with simple harmonic motion from its mean position measured in the same direction are 1, 5 and 5. Show that the period of a complete oscillation is $2\pi/\cos^{-1}(3/5)$. 6

(D) Show that the particle executes S.H.M. requires $1/6^{th}$ of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude. 6

**UNIT—III**

3. (A) Derive Lagrange's equations of motion for conservative holonomic system as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1, 2, ..., n.$$

where the quantity L = T – V is Lagrangian of the system. 6

(B) Two particles of masses $m_1$ and $m_2$ are connected by a light inextensible string which passes over a small smooth fixed pulley. If $m_1 > m_2$, then show that the common acceleration of the particle is $\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$. 6

**OR**

(C) Derive Lagrange's equations of motion in the form :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q'_j, \quad j = 1, 2, ..., n$$

for a partly conservative system, where the quantity L refers to the conservative part of the system and $Q'_j$ refers to the forces which are not conservative. 6

(D) Prove that Lagrange's equations of motion takes the form :

$$\left(\frac{\partial L}{\partial \dot{q}_j}\right)^{\bullet} - \frac{\partial L}{\partial q_j} + \frac{\partial R}{\partial \dot{q}_j} = 0, \quad j = 1, 2, ..., n,$$

when the frictional forces acting on the system are such that they are derivable in terms of Rayleigh's dissipation function R. 6

**UNIT—IV**

4. (A) Prove that the problem of motion of two masses interacting with only one another can always be reduced to a problem of the motion of a single mass. 6

(B) For a system moving in a finite region of space with finite velocity, prove that the time average of kinetic energy is equal to the virial of the system. 6

**OR**

(C) In a central force field, prove that :

(i) The path of a particle lies in one plane and

(ii) The areal velocity is conserved. 6

(D) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on the circle, then the force varies as the inverse fifth power of the distance. 6

## QUESTION—V

5. (A) If two forces P and Q are acting at two different points of a rigid body. Then define : like parallel forces, unlike parallel forces and also state when do they form a couple. 1½

(B) For a common catenary, prove that y = c sec $\psi$. 1½

(C) Prove the relation : $\frac{d\hat{n}}{dt} = -\left(\frac{d\theta}{dt}\right)\hat{r}$, where $\hat{r}$ and $\hat{n}$ are unit vectors along and perpendicular to the radial direction respectively in XY-plane and $\theta$ is the angle made by radius vector with an axis OX. 1½

(D) A point describes a cycloid s = 4a sin $\psi$ with uniform speed $v$. Show that its tangential accleration is zero. 1½

(E) Define : Holonomic and Non-Holonomic constraint. 1½

(F) State D'Alembert's principle for a mechanical system of n particles. 1½

(G) If the total torque on a particle is zero, then show that the angular momentum is conserved. 1½

(H) For an inverse square law, show that virial theorem takes a form $\overline{T} = -\frac{1}{2}V$. 1½